Enrollment No: ______________________ Exam Seat No:______________________

C.U.SHAH UNIVERSITY

Summer Examination-2018

Subject Name: Topology

Subject Code: 4SC06TOC1 **Branch: B.Sc. (Mathematics)**

Semester: 6 **Date: 02/05/2018** **Time: 02:30 To 05:30** **Marks : 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) Define: Topology. **(01)**
b) Let X be any set, write co-countable topology on X. **(01)**
c) Define:Lower limit topology on R **(01)**
d) Consider R with usual topology and lower limit topology. Which topology is stronger than the other? **(01)**
e) Define: Door space. **(01)**
f) Define: Interior of set. **(01)**
g) Define: Closure of set. **(01)**
h) Is it true that $(A \cap B)' = A' \cap B'$? **(01)**
i) Define: Boundary point of set. **(01)**
j) Define: Continuous function. **(01)**
k) Define: Homeomorphism. **(01)**
l) Define: Disconnected space **(01)**
m) Define: T_2 space. **(01)**
n) Define: Compact space. **(01)**

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

a) Let X be a set. **(05)**
Let $\tau_f = \{U \subset X \mid \text{either} X - U = X \text{or} X - U \text{is finite}\}$then prove that (X, τ_f) is topological space.

b) Let X be topological space. Then prove that, **(05)**
(i) $\emptyset$, X are closed
(ii) Any finite union of closed sets in $X$is closed.
(iii) Any arbitrary intersection of closed sets in X is closed.

c) Let X be a non-empty set. Compare the following topology,
(i) Co countable topology (ii) Co – finite topology (iii) Indiscrete topology **(04)**
(iv) Discrete topology

Q-3 **Attempt all questions** **(14)**

a) Let (X, τ) be a topological space &A, B be two subsets of X then prove that, **(07)**
(i) If $A \subset B$ then $A^0 \subset B^0$.

(ii) $(A \cap B)^0 = A^0 \cap B^0$.
(iii) $A^0 \cup B^0 \subset (A \cup B)^0$. Give an example which shows that

$$A^0 \cup B^0 \neq (A \cup B)^0$$

b) Let (X, τ) be a topological space and Y be a non empty subset of X then prove that the collection $\tau_y = \{U \cap Y \mid U \in \tau\}$ is a topology on Y. **(05)**

c) Let $X = R$ and $A = (0,1)$, find $A^0, \bar{A}, ext\ (A)$and A'. **(02)**

Q-4 **Attempt all questions** **(14)**

a) Let X be a topological space and A be a subset of X. Then prove that $\bar{A} = A \cup A^{'}$. **(05)**

b) Let X, Y, Z be topology space and $f: X \rightarrow Y$ and $g: Y \rightarrow Z$ are continuous functions then prove that $gof: X \rightarrow Z$ is continuous. **(05)**

c) Let (X, τ) be disconnected space and $\tau^{'}$ is finer than τ. Prove that $(X, \tau^{'})$ is disconnected. **(04)**

Q-5 **Attempt all questions** **(14)**

a) Let X and Y be a topological space and $f: X \rightarrow Y$, then prove that following are equivalent **(06)**
(i) f is continuous.
(ii) For every subset A of X then $f(\bar{A}) \subset \overline{f(A)}$.
(iii) For every close set B in Y then $f^{-1}(B)$ is closed in X.

b) Prove that R with lower limit topology is disconnected. **(04)**

c) If X has more than two elements then prove that X is T_2 space with discrete topology. **(04)**

Q-6 **Attempt all questions** **(14)**

a) Prove that every closed subset of compact space is compact. **(05)**

b) Prove that continuous image of connected space is connected. **(05)**

c) Prove that every subspace of T_1 space is T_1 space. **(04)**

Q-7 **Attempt all questions** **(14)**

a) Prove that every compact subset of T_2 space is closed. **(06)**

b) Prove that continuous image of compact space is compact. **(05)**

c) Is R compact space with usual topology? Justify your answer. **(03)**

Q-8 **Attempt all questions** **(14)**

a) State and prove Heine Borel theorem. **(09)**

b) Give an example of topological space which is T_1 space but not T_2 space. **(05)**